In memory of Fedir Semenovych Rofe-Beketov (1932–2022)

Fedir Semenovych Rofe-Beketov was born on December 24, 1932 in Kharkiv in the well-known Beketov-Alchevsky family having ties to A. Blok, D.I. Mendeleev, geologist D.I. Shcherbakov and other outstanding personalities of culture and science.

In 1955, he graduated from Kharkiv State University. He completed his first two scientific works while still being an undergraduate student. From 1955 to 1958 he was a post-graduate student at Kharkiv University, being supervised by V.O. Marchenko. In 1962, he defended his Ph.D. thesis "Some problems of spectral analysis of infinite systems of differential equations", and in 1987, he was awarded Doctor of Science degree for the thesis "Problems of extension theory and spectral analysis of linear differential operators".

F.S. Rofe-Beketov's professional activity was affiliated with the Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine. His research interests focus on spectral theory for differential operators. The first works of F.S. Rofe-Beketov in this area were devoted to the construction of eigenfunction expansions for infinite systems of differential equations in the self-adjoint and non-self-adjoint cases, and the inverse problem for such systems on the half-line. Then he turned to the study of the spectrum of differential operators with periodic coefficients, where he obtained several important results aimed at answering the question posed by I.M. Gelfand in 1957 on the topology of the spectrum of a differential equation with complex periodic coefficients. During this period, he also solved the problem posed by M.Sh. Birman and I.M. Glazman and concerning conditions for the finiteness of the number of discrete levels appearing in the gaps of the continuous spectrum due to a perturbation of the periodic potential. He presented these results at the International

Congress of Mathematicians in 1966. Later F.S. returned to this problem and investigated perturbations of the spectrum of almost periodic operators. He introduced and calculated Kneser-type constants for each spectral gap. Also, jointly with his student V.I. Khrabustovskyi, F.S. studied stability of infinite periodic systems, where they discovered a relationship between stability and integrability properties of operator coefficients.

During the 1960th, F.S. Rofe-Beketov also gave a complete solution to the inverse problem for a one-dimensional Schrödinger operator on the whole axis with an arbitrary real potential. The necessary and sufficient conditions on the spectral matrices of such an operator, obtained by F.S., have found various applications and are widely used. At the same time, jointly with his student E.Kh. Khristov, he studied the inverse scattering problem for the Sturm–Liouville operator with a strongly singular potential.

In the late 1960s, F.S. Rofe-Beketov turned to extension theory for differential operators. He was the first who utilized the effective method of binary relations in the extension theory. Using this method, he described all self-adjoint extensions for differential operators (of arbitrary order) with bounded operator-valued coefficients on a finite interval in terms of boundary conditions. Nowadays, this method has numerous generalizations and applications, and is presented in a lot of monographs, articles, and reviews.

Another area of Rofe-Beketov's research in extension theory was the problem of essential self-adjointness of differential operators. In the works carried out by him personally and also jointly with his student O.G. Brusentsev, a number of strong criteria for the essential self-adjointness of strongly elliptic operators were obtained. Jointly with H. Kalf, he investigated the essential self-adjointness of a non-semibounded Schrödinger operator with a locally integrable potential. Jointly with his student V.I. Kogan, he obtained important results on the possible values of the defect indices of symmetric differential operators with complex coefficients, as well as on the formal defect numbers of symmetric systems with degenerate weight.

In the late 1970s and in the 1980s, F.S. carried out important investigations for applications on perturbations of the spectrum of semibounded operators related to changing operator domains and strong resolvent convergence. At the same period, F.S. Rofe-Beketov was attracted by the problem to find the relationship between spectral and oscillatory properties of differential equations of arbitrary even order with bounded operator coefficients. Jointly with his student O.M. Kholkin, he was the first who constructed an oscillation theory for such differential equations, covering, in particular, the classical Sturm theorem and a number of its well-known generalizations, including the Morse index theorem. The results of these and a lot of other investigations appeared in joint monographs with O.M. Kholkin: "Spectral analysis of differential operators (connection between spectral and oscillatory properties)" (2001) and "Spectral analysis of differential operators. Interplay between spectral and oscillatory properties" (2005).

In 2000, jointly with his student O.I. Bondarenko, F.S. Rofe-Beketov solved

the inverse scattering problem on the axis and semiaxis for Sturm–Liouville systems with a triangular matrix potential. Later, for the first time for non-self-adjoint operators, jointly with O.M. Kholkin he established a connection between spectral and oscillatory properties for systems with block-triangular matrix coefficients.

F.S. Rofe-Beketov devoted much time to teaching. His lectures were thoughtfully and carefully prepared. He was always attentive and exacting to his students. He supervised six Ph.D. students, four of whom later got the title Doctor of Science. F.S. was a member of editorial boards of three mathematical journals, including our "Journal of Mathematical Physics, Analysis, Geometry".

For his outstanding scientific achievements, he was awarded the Ostrogradsky prize and the award "For educating young generation" of the National Academy of Sciences of Ukraine, as well as Bogolyubov memorial medal.

Fedir Semenovych was a person of great culture and erudition. He was vividly interested in history, literature, architecture, painting and music. Also, Fedir Semenovych worked a lot and fruitfully on the history of the Beketov and Alchevsky families, whose activities are closely connected with the history of the city of Kharkiv, and with the world science and culture.

Fedir Semenovych died on April 28, 2022. We will always carry the memory of Fedir Semenovych Rofe-Beketov with us.

G. Feldman, O. Kholkin, V. Khrabustovskyi,
E. Khruslov, V. Kotlyarov, V. Marchenko,
L. Pastur, M. Shcherbina