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On the Long-Time Behavior of the Thermoelastic Plates with Second Sound

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The relation between the thermoelastic Cattaneo model and the thermoelastic Gurtin–Pipkin model is established. The existence of the compact global attractor of the Cattaneo–Mindlin plate model is proved and its properties are studied.

Key words: second sound, asymptotic behavior, global attractor.

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1. Introduction and Functional Settings

In this paper we consider the nonlinear Mindlin–Timoshenko model of a thermoelastic plate with heat conduction of Cattaneo type (for review see [1]). The Mindlin–Timoshenko model describes dynamics of a plate in view of transverse shear effects (see, e.g., [2, 3] and references therein). Unlike the classical Fourier constitutive law, the Cattaneo model describes heat conduction processes under the assumption of finite speed propagation of disturbances.

We assume that the plate has a uniform thickness h and, when in equilibrium, its middle surface lies in the bounded domain $\Omega \subset (x_1, x_2, 0)$ with the sufficiently smooth boundary $\partial\Omega$. The integro-differential equations for the vector of angles of deflection of the filament $v(x,t) = (v_1(x,t), v_2(x,t)) \in \mathbb{R}^2$, the transverse displacement of the middle surface $w(x,t) \in \mathbb{R}$, the temperature variation $\theta(x,t)$ and the heat flax $q(x,t)$ averaged with respect to the thickness, where $x = (x_1, x_2) \in \Omega$ and $t \geq 0$, are the following:

$$\begin{aligned} &\alpha_0 v_{tt} + \beta_0 v_t - \mathcal{A} v + \mu(v + \nabla w) + \beta \nabla\theta + \nabla_v \Phi(v) = 0, \\ &\alpha_1 w_{tt} + \beta_1 w_t - \mu \mathrm{div}(v + \nabla w) + g(w) = 0, \\ &\gamma\theta_t + \kappa \mathrm{div}\mathbf{q} + \beta \mathrm{div} v_t = 0, \\ &\omega \mathbf{q}_t + \mathbf{q} + \nabla\theta = 0. \end{aligned} \tag{1}$$

Here the vector function $\nabla_v \Phi(v) = (\partial_{v_1}\Phi(v_1, v_2), \partial_{v_2}\Phi(v_1, v_2))$ and the scalar function $g(w)$ are the feedback forcing terms. The parameters α_0, α_1, β_0, β_1, β, γ, κ, μ, ω are positive constants. The operator $\mathcal{A}$ has the structure

$$\mathcal{A} = \begin{pmatrix} \partial^2_{x_1} + \frac{1-\nu}{2}\partial^2_{x_2} & \frac{1+\nu}{2}\partial_{x_1 x_2} \\ \frac{1+\nu}{2}\partial_{x_1 x_2} & \frac{1-\nu}{2}\partial^2_{x_1} + \partial^2_{x_2} \end{pmatrix} = \nabla \mathrm{div} - \frac{1-\nu}{2}\mathrm{rotrot},$$

where $0 < \nu < 1$ is the viscoelastic Poisson's ratio.

The Timoshenko systems have been treated by many authors. There are several works investigating the presence or the lack of the exponential stability of linear and nonlinear problems with various types of damping and boundary conditions for the Timoshenko (see, e.g., [4, 5]) and Timoshenko–Cattaneo problems [6, 7]. The existence and the properties of attractors for the related systems were established in [8–10]. In [8], the existence of a compact global attractor for the Mindlin–Timoshenko elasticity and its upper semicontinuity, as the shear modulus tends to infinity, are shown. Paper [10] is devoted to the existence of a compact global attractor and its properties of the Mindlin–Timoshenko viscoelastic system of memory type coupled with Gurtin–Pipkin heat conduction equations (see [11] for the model description). The long-time behavior of the Mindlin–Timoshenko problem

$$\begin{aligned} &\alpha_0 v_{tt} + \beta_0 v_t - \mathcal{A}v + \mu(v + \nabla w) + \beta\nabla\theta + \nabla_v \Phi(v) = 0, \\ &\alpha_1 w_{tt} + \beta_1 w_t - \mu \mathrm{div}(v + \nabla w) + g(w) = 0, \\ &\gamma\theta_t - \tfrac{1}{\omega^2}\int_0^\infty \eta(\tfrac{s}{\omega})\Delta\tau(s)ds + \beta \mathrm{div} v_t = 0, \\ &\theta = \tau_t + \tau_s, \quad s \ge 0 \end{aligned} \tag{2}$$

for the model with Gurtin–Pipkin heat conduction with Dirichlet boundary conditions was studied in [9].

The main goal of the paper is to study the long-time behavior of the semilinear thermoelastic Mindlin–Timoshenko–Cattaneo system with locally Lipschitz nonlinearities of any polynomial growth (of odd degrees) and to establish the closeness of the family of attractors to the attractor of the Fourier thermoelastic model in a suitable sense in limit case $\omega \to 0$. In the present paper, we establish the relation between the dynamics of problems (1) and (2). It is shown that (1) can be decomposed into two systems. The energy of the first system decays exponentially to zero. The dynamics of the second one is connected with the dynamics of system (2) according to the law given in Lemma 2. Additionally, we describe the relation between the structures of the attractors of systems (1) and (2). We also establish the upper-semicontinuity of the family of attractors of (1) with respect to the relaxation time.

Rewrite the thermoelastic model with second sound (1) in the following way:

$$\begin{aligned} &Pu_{tt} + Mu_t + Au + R\theta = F(u), \\ &\gamma\theta_t + \kappa \mathrm{div}\mathbf{q} + \beta \mathrm{div} v_t = 0, \\ &\omega \mathbf{q}_t + \mathbf{q} + \nabla\theta = 0, \end{aligned} \tag{3}$$

and subject it to the initial and boundary conditions

$$\begin{array}{lll} u(x,0) = u_0(x) \in [H_0^1(\Omega)]^3, & u_t(x,0) = u_1(x) \in [L^2(\Omega)]^3, & x \in \Omega, \\ \theta(x,0) = \theta_0(x) \in L^2(\Omega), & \mathbf{q}(x,0) = \mathbf{q}_0(x) \in [L^2(\Omega)]^2, & x \in \Omega, \\ \theta(x,t) = 0, & u(x,t) = 0, \quad x \in \partial\Omega, t \geq 0. & \end{array} \tag{4}$$

The operator A with the domain

$$\mathcal{D}(A) = \{u = (v_1, v_2, w) \in [(H^2 \cap H_0^1)(\Omega)]^3\}$$

has the structure

$$A = \begin{pmatrix} -\mathcal{A} + \mu I & \mu\nabla \\ -\mu \mathrm{div} & -\mu\Delta \end{pmatrix}.$$

Obviously, A is a positive self-adjoint operator with the square root possessing the domain $\mathcal{D}(A^{1/2}) = [H_0^1(\Omega)]^3$. It is easy to see that the operators

$$\{R : H_0^1(\Omega) \to [L^2(\Omega)]^3,\, R\theta = \beta(\partial_1\theta, \partial_2\theta, 0)\}$$

and

$$\{Q : [H_0^1(\Omega)]^3 \to L^2(\Omega),\, Q\mathtt{u} = -\beta(\partial_1 \mathtt{u}_1 + \partial_2 \mathtt{u}_2), \;\; \mathtt{u} = (\mathtt{u}_1, \mathtt{u}_2, \mathtt{u}_3)\}$$

possess the property $(R\theta, \mathtt{u}) = (\theta, Q\mathtt{u})$ for any $\theta \in H_0^1(\Omega)$ and $\mathtt{u} \in [H_0^1(\Omega)]^3$. The bounded in $[L^2(\Omega)]^3$ operators P and M are defined by the formulas

$$P = \begin{pmatrix} \alpha_0 I & 0 \\ 0 & \alpha_1 I \end{pmatrix}, \qquad M = \begin{pmatrix} \beta_0 I & 0 \\ 0 & \beta_1 I \end{pmatrix}.$$

The nonlinear term has the structure

$$F(u) = \begin{bmatrix} -\partial_{v_1}\Phi(v_1, v_2) \\ -\partial_{v_2}\Phi(v_1, v_2) \\ -g(w) \end{bmatrix}, \quad u = (v_1, v_2, w). \tag{5}$$

We assume that the memory kernel $\eta(s)$ possesses the properties

$$\eta \in C^1(\mathbb{R}^+) \cap L^1(\mathbb{R}^+), \tag{6}$$

moreover,

$$\eta(s) \geq 0, \tag{7}$$

and for any $s \in \mathbb{R}^+$ there exists $l > 0$ such that for any $s \in \mathbb{R}^+$

$$\eta'(s) + l\eta(s) \leq 0. \tag{8}$$

We will use the notation

$$\eta_\omega(s) = \frac{1}{\omega^2}\eta(\frac{s}{\omega}). \tag{9}$$

Introduce a Hilbert space $L^2_\omega(\mathbb{R}^+, H^l(\Omega))$ of H^l-valued functions on $\mathbb{R}^+$ $(l \in \mathbb{R})$ such that

$$\|\tau\|^2_{L^2_\omega(\mathbb{R}^+, H^l(\Omega))} \equiv \omega \int_0^\infty \eta_\omega(s)\|\tau(s)\|^2_{H^l(\Omega)} ds < \infty,$$

which denotes a norm in this space. We endow $L^2_\omega(\mathbb{R}^+, H^l(\Omega))$ with the inner product

$$(\phi_1, \phi_2)_{L^2_\omega(\mathbb{R}^+, H^l(\Omega))} = \omega \int_0^\infty \eta_\omega(s)(\phi_1(s), \phi_2(s))_{H^l(\Omega)} ds.$$

We will also need the space

$$H^1_\omega(\mathbb{R}^+, H^1_0(\Omega)) = \{\phi : \phi(s), \phi_s(s) \in L^2_\omega(\mathbb{R}^+, H^1_0(\Omega))\}.$$

Define the operator $T_\omega : \mathcal{D}(T_\omega) \to L^2(\Omega)$ with the domain

$$\mathcal{D}(T_\omega) = \{\tau \in L^2_\omega(\mathbb{R}^+, H^1_0(\Omega)) : \int_0^\infty \eta_\omega(s)\Delta\tau ds \in L^2(\Omega), \tau|_{s=0} = 0\}$$

by the formula

$$T_\omega\tau = -\int_0^\infty \eta_\omega(s)\Delta\tau ds.$$

We assume that the nonlinearities of the problem satisfy the conditions

$$g \in C^1(\mathbb{R}), \quad \Phi \in C^2(\mathbb{R}^2), \tag{10}$$

and there exist $q > 0$ and $\tilde{C} > 0$ such that

$$\begin{gathered} |g'(z)| \leq \tilde{C}(1 + |z|^q), \\ |\partial_1^2\Phi(z)| + |\partial_2^2\Phi(z)| + |\partial_1\partial_2\Phi(z)| \leq \tilde{C}(1 + |z|^q). \end{gathered} \tag{11}$$

Moreover, there exist $b_i \in \mathbb{R},\ i = 1, 2$ such that

$$\begin{gathered} \Phi(z_1, z_2) \geq -b_1, \\ G(z) \equiv \int_0^z g(\zeta)d\zeta \geq -b_2. \end{gathered} \tag{12}$$

We also assume that there exist $a_i > 0, i = \overline{1,4}$ such that

$$\begin{array}{c} -a_1\Phi(z) + \nabla_z\Phi(z)\, z \geq -a_2, \\ -a_3 G(z) + g(z)\, z \geq -a_4. \end{array} \tag{13}$$

In the paper we will establish the relation between problem (3)–(4) and the Gurtin–Pipkin thermoelastic model considered in [9]:

$$Pu_{tt} + Mu_t + Au + R\theta = F(u), \tag{14}$$

$$\gamma\theta_t + T_\omega\tau - Qu_t = 0, \tag{15}$$

$$\theta = \tau_t + \tau_s, \qquad s \geq 0 \tag{16}$$

$$u|_{t=0} = u_0, \;\; u_t|_{t=0} = u_1, \;\; \theta|_{t=0} = \theta_0, \;\; \tau(t,s)|_{t=0} = \tau_0(s), \;\; s \geq 0. \tag{17}$$

Introduce the space

$$X_\omega = \mathcal{D}(A^{1/2}) \times [L^2(\Omega)]^3 \times L^2(\Omega) \times [L^2(\Omega)]^2$$

with the inner product

$$\left(\begin{bmatrix} u_1 \\ \mathtt{v}_1 \\ \theta_1 \\ \mathbf{q}_1 \end{bmatrix}, \begin{bmatrix} u_2 \\ \mathtt{v}_2 \\ \theta_2 \\ \mathbf{q}_2 \end{bmatrix} \right)_{X_\omega} = (A^{1/2}u_1, A^{1/2}u_2) + (P^{1/2}\mathtt{v}_1, P^{1/2}\mathtt{v}_2) + \gamma(\theta_1, \theta_2) + \omega\kappa(\mathbf{q}_1, \mathbf{q}_2)$$

and the space

$$\tilde{D} = \{\phi \in [L^2(\Omega)]^2 : \mathrm{div}\phi \in L^2(\Omega)\}$$

with the inner product

$$(\mathbf{q}_1, \mathbf{q}_2)_{\tilde{D}} = (\mathrm{div}\mathbf{q}_1, \mathrm{div}\mathbf{q}_2) + (\mathbf{q}_1, \mathbf{q}_2).$$

We define the operator $\mathbb{B}_\omega : X_\omega \supset \mathcal{D}(\mathbb{B}_\omega) \to X_\omega$ with the domain

$$\mathcal{D}(\mathbb{B}_\omega) = \{(u, \mathtt{u}, \theta, \mathbf{q}) : u \in \mathcal{D}(A), \mathtt{u} \in \mathcal{D}(A^{1/2}), \theta \in H_0^1(\Omega), \mathbf{q} \in \tilde{D}\}$$

by the formula

$$\mathbb{B}_\omega = \begin{pmatrix} 0 & I & 0 & 0 \\ -P^{-1}A & -P^{-1}M & -P^{-1}R & 0 \\ 0 & \frac{1}{\gamma}Q & 0 & -\frac{\kappa}{\gamma}\mathrm{div} \\ 0 & 0 & -\frac{1}{\omega}\nabla & -\frac{1}{\omega} \end{pmatrix}.$$

We will use the notations $Z(t) = (u(t), \mathtt{u}(t), \theta(t), \mathbf{q}(t))$ and $Z_0 = (u_0, u_1, \theta_0, \mathbf{q}_0) \in X_\omega$, where $\mathtt{u}(t) = u_t(t)$. Then the problem (3)–(4) can be rewritten as follows:

$$\begin{array}{c} \frac{d}{dt}Z(t) - \mathbb{B}_\omega Z(t) = f(Z(t)), \\ Z(0) = Z_0. \end{array} \tag{18}$$

Introduce the space

$$H_\omega = \mathcal{D}(A^{1/2}) \times [L^2(\Omega)]^3 \times L^2(\Omega) \times L^2_\omega(\mathbb{R}^+, H^1_0(\Omega))$$

with the inner product

$$\left(\begin{bmatrix} u_1 \\ \mathtt{v}_1 \\ \theta_1 \\ \tau_1 \end{bmatrix}, \begin{bmatrix} u_2 \\ \mathtt{v}_2 \\ \theta_2 \\ \tau_2 \end{bmatrix} \right)_{H_\omega} = (A^{1/2}u_1, A^{1/2}u_2) + (\mathtt{v}_1, \mathtt{v}_2) + \gamma(\theta_1, \theta_2) + \frac{1}{\omega}\langle \nabla\tau_1, \nabla\tau_2 \rangle.$$

Denote $\mathbb{A}^\eta_\omega : \mathcal{D}(\mathbb{A}^\eta_\omega) \to H_\omega$ to be

$$\mathbb{A}^\eta_\omega = \begin{pmatrix} 0 & I & 0 & 0 \\ -A & -D & -R & 0 \\ 0 & \frac{1}{\gamma}Q & 0 & \frac{1}{\gamma}T_\omega \\ 0 & 0 & I & -\partial_s \end{pmatrix}$$

with the domain

$$\mathcal{D}(\mathbb{A}^\eta_\omega) = \mathcal{D}(A) \times \mathcal{D}(A^{1/2}) \times H^1_0(\Omega) \times [\mathcal{D}(T_\omega) \cap H^1_\omega(\mathbb{R}^+, H^1_0(\Omega))].$$

For problem (14)–(17), we use the result obtained in [9]:

Proposition 1. *The operator $\mathbb{A}^\eta_\omega$ is the generator of the C_0-semigroup $U^\eta_\omega(t)$ on the space H_ω.*

Relying on this result we give the definition of the mild solution to (14)–(17).

Definition 1. *The function $Z = (u, u_t, \theta, \tau) \in C(0, T; H_\omega)$ is a mild solution to (14)–(17) on the interval $[0, T]$ subjected to the initial conditions $Z(0) = Z_0 = (u_0, u_1, \theta_0, \tau_0)$ if the relation*

$$S_\omega(t)Z_0 = Z(t) = U^\eta_\omega(t)Z_0 + \int_0^t U^\eta_\omega(t-s)f(Z(s))ds$$

holds true for any $t \in [0, T]$.

The long-time behavior of the dynamical system generated by problem (14)–(17), i.e., the existence and the properties of the compact global attractor, was studied in paper [9]. By definition (see, e.g., [12, 13]), a global attractor is a bounded closed set $\mathfrak{A}_\omega \subset H_\omega$ such that $S_\omega(t)\mathfrak{A}_\omega = \mathfrak{A}_\omega$ for all $t \geq 0$, and

$$\lim_{t\to+\infty} \sup_{y\in B} \operatorname{dist}(S(t)y, \mathfrak{A}_\omega) = 0$$

for any bounded set $B \subset H_\omega$.

We recall here the theorem proven in [9].

Theorem 1. *Let the assumptions* (6)–(8) *and* (10)–(13) *hold. Then for any* $\omega > 0$ *the dynamical system* $(S_\omega(t), H_\omega)$ *generated by* (14)–(17) *possesses a compact global attractor* $\mathfrak{A}_\omega$ *whose fractal dimension is finite. The family of attractors* $\{\mathfrak{A}_\omega\}$ *is upper semicontinuous at zero, i.e.,*

$$\sup_{y\in\mathfrak{A}_\omega} \operatorname{dist}_{H_\omega}(y, \mathfrak{A}_0) \to 0, \quad \omega \to 0,$$

where

$$\mathfrak{A}_0 = \left\{ y = \begin{bmatrix} u_0 \\ u_1 \\ \theta_0 \\ 0 \end{bmatrix} : \begin{bmatrix} u_0 \\ u_1 \\ \theta_0 \end{bmatrix} \in \mathfrak{A} \right\},$$

where $\mathfrak{A}$ *is the compact global attractor of the dynamical system generated by the problem*

$$\begin{aligned} &Pu_{tt} + Mu_t + Au + R\theta = F(u), \\ &\gamma\theta_t - \kappa\Delta\theta - Qu_t = 0, \\ &u|_{t=0} = u_0, \quad u_t|_{t=0} = u_1, \quad \theta|_{t=0} = \theta_0 \end{aligned} \tag{19}$$

in the space $[H_0^1(\Omega)]^3 \times [L^2(\Omega)]^2 \times L^2(\Omega)$.

2. Long-Time Behavior

To study the dynamics of Mindlin–Timoshenko–Cattaneo system (3)–(4), we consider the problems

$$\begin{gathered} Pu_{tt} + Mu_t + Au + R\theta = F(u), \\ \gamma\theta_t + \kappa\Delta q + \beta \operatorname{div} v_t = 0, \quad x \in \Omega, \quad t > 0, \\ \omega q_t + q + \theta = 0, \\ u(x,0) = u_0(x) \in [H_0^1(\Omega)]^3, \quad u_t(x,0) = u_1(x) \in [L^2(\Omega)]^3, \\ \theta(x,0) = \theta_0(x) \in L^2(\Omega), \quad q(x,0) = q_0(x) \in H_0^1(\Omega) \end{gathered} \tag{20}$$

and

$$\begin{gathered} \omega p_t + p = 0, \\ p(0) = p_0 \in [L^2(\Omega)]^2. \end{gathered} \tag{21}$$

Define the space

$$\mathcal{V}_\omega = \mathcal{D}(A^{1/2}) \times [L^2(\Omega)]^3 \times L^2(\Omega) \times H_0^1(\Omega)$$

with the norm

$$\|(u, \mathrm{v}, \theta, q)\|^2_{\mathcal{V}_\omega} = \|A^{1/2}u\|^2 + \|P^{1/2}\mathrm{v}\|^2 + \gamma\|\theta\|^2 + \omega\kappa\|\nabla q\|^2.$$

Obviously problem (21) generates an exponentially stable dynamical system in the space $[L^2(\Omega)]^2$, moreover, $p = p_0 e^{-\frac{t}{\omega}}$ tends to zero as $\omega \to 0$.

Problem (20) can be rewritten as (14)–(17) with the kernel $\eta(s) = \kappa e^{-s}$ if we prolong θ in (20) backward in time, for instance, in the following way:

$$\theta(x, -t) = -q_0(x), \quad t > 0.$$

It follows from the last equation in (20) that

$$q(x,t) = e^{-\frac{t-\xi}{\omega}} q_0(x) - \frac{1}{\omega} \int\limits_{\xi}^{t} e^{-\frac{(t-s)}{\omega}} \theta(x,s) ds, \quad \xi \leq t, \ \ t \geq 0.$$

Consequently, letting ξ tend to $-\infty$ and changing variables, we get

$$q(x,t) = -\frac{1}{\omega^2} \int\limits_0^{\infty} e^{-\frac{s}{\omega}} \tau(x,t,s) ds. \tag{22}$$

Substituting (22) into (20) and defining

$$\tau_0 = -s q_0(x) \in L^2_\omega(\mathbb{R}^+, H^1_0(\Omega)),$$

we arrive at problem (14)–(17). It is easy to see that the kernel $\eta(s) = \kappa e^{-s}$ satisfies conditions (6)–(8), and $\eta_0 = \eta_1 = \kappa$. Consequently, Theorem 1 is valid for the obtained problem of the kind (14)–(17). To establish the results analogous to those stated in Theorem 1 for problem (3)–(4), we have to study the relation between problems (14)–(17) and (20). At first we will show that the existence of the dynamical system $(S_\omega(t), H_\omega)$ generated by problem (14)–(17) with the kernel $\eta(s) = \kappa e^{-s}$ leads to the existence of the C_0-semigroup $\Sigma_\omega(t)$ on the space $\mathcal{V}_\omega$ generated by the mild solutions of problem (20).

To give the definition of the solutions, we introduce the operators $\mathcal{K} : H^1_0(\Omega) \to L^2_\omega(\mathbb{R}^+, H^1_0(\Omega))$ and $\mathcal{N} : L^2_\omega(\mathbb{R}^+, H^1_0(\Omega)) \to H^1_0(\Omega)$ acting by the formulas

$$\mathcal{K} q = -sq \quad \text{and} \quad \mathcal{N}\tau = -\frac{1}{\omega^2} \int\limits_0^{\infty} e^{-\frac{s}{\omega}} \tau ds.$$

The operators $\mathfrak{K} : \mathcal{V}_\omega \to H_\omega$ and $\mathfrak{N} : H_\omega \to \mathcal{V}_\omega$ are defined by

$$\mathfrak{K} = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & \mathcal{K} \end{pmatrix}, \qquad \mathfrak{N} = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & \mathcal{N} \end{pmatrix}.$$

Let the operator $\mathbb{D}_\omega : \mathcal{V}_\omega \supset \mathcal{D}(\mathbb{D}_\omega) \to \mathcal{V}_\omega$ with the domain

$$\mathcal{D}(\mathbb{D}_\omega) = \{(u, \mathrm{u}, \theta, q) : u \in \mathcal{D}(A), \mathrm{u} \in \mathcal{D}(A^{1/2}), \theta \in H_0^1(\Omega), \mathbf{q} \in H^2 \cap H_0^1(\Omega)\}$$

have the structure

$$\mathbb{D}_\omega = \begin{pmatrix} 0 & I & 0 & 0 \\ -P^{-1}A & -P^{-1}M & -P^{-1}R & 0 \\ 0 & \frac{1}{\gamma}Q & 0 & -\frac{\kappa}{\gamma}\Delta \\ 0 & 0 & -\frac{\kappa}{\omega} & -\frac{1}{\omega} \end{pmatrix}.$$

The operators $\mathfrak{N}$ and $\mathfrak{K}$ possess the following properties.

Proposition 2. *For any $Z \in \mathcal{D}(\mathbb{D}_\omega)$,*

$$\mathfrak{N}\mathbb{A}_\omega\mathfrak{K}Z = \mathbb{D}_\omega Z, \tag{23}$$

where $\mathbb{A}_\omega$ is the generator of the semigroup generated by problem (14)–(17) *with the kernel $\eta(s) = \kappa e^{-s}$. Moreover,*

$$\mathfrak{N}\mathfrak{K} = I \in \mathfrak{L}(\mathcal{V}_\omega). \tag{24}$$

P r o o f. It is obvious that if $Z \in \mathcal{D}(\mathbb{D}_\omega)$, then $\mathfrak{K}Z \in \mathcal{D}(\mathbb{A}_\omega)$. Therefore the statement of the lemma can be easily proved by the straightforward calculations.

Let $Z(t) = (u(t), \mathrm{u}(t), \theta(t), q(t))$ and $Z_0 = (u_0, u_1, \theta_0, q_0) \in \mathcal{V}_\omega$, where $\mathrm{u}(t) = u_t(t)$. Then problem (20) can be rewritten as follows:

$$\begin{aligned} \tfrac{d}{dt}Z(t) - \mathbb{D}_\omega Z(t) &= f(Z(t)), \\ Z(0) &= Z_0. \end{aligned} \tag{25}$$

Lemma 1. *The operator $\mathbb{D}_\omega$ is the generator of the C_0-semigroup $\mathcal{W}_\omega(t) = \mathfrak{N}U_\omega(t)\mathfrak{K}$ on the space $\mathcal{V}_\omega$, where $U_\omega(t)$ is the exponentially stable C_0-semigroup on the space H_ω generated by the operator $\mathbb{A}_\omega$.*

P r o o f. Consider the operator $\mathcal{W}_\omega(t) = \mathfrak{N}U_\omega(t)\mathfrak{K}$. To prove that $\mathcal{W}_\omega(t)$ is a C_0-semigroup on the space $\mathcal{V}_\omega$, we have to check the semigroup properties. It follows from (24) that

$$\mathcal{W}_\omega(0) = \mathfrak{N}U_\omega(0)\mathfrak{K} = \mathfrak{N}\mathfrak{K} = I.$$

Now we prove that

$$\mathcal{W}_\omega(r)\mathcal{W}_\omega(t) = \mathfrak{N}U_\omega(r)\mathfrak{K}\mathfrak{N}U_\omega(t)\mathfrak{K} = \mathfrak{N}U_\omega(t+r)\mathfrak{K} = \mathcal{W}_\omega(t+r). \tag{26}$$

Define the operators $U_i(t), i = \overline{1,4}$ as follows:

$$U_1(t)Z_0 = u(t),\; U_2(t)Z_0 = u_t(t),\; U_3(t)Z_0 = \theta(t),\; U_4(t)Z_0 = \tau(t),$$

where $(u(t), u_t(t), \theta(t), \tau(t))$ is a mild solution to problem (14)–(17) with initial conditions Z_0. Note that $U_\omega(t) = (U_1(t), U_2(t), U_3(t), U_4(t))$.

Point out that

$$\begin{aligned}\mathcal{W}_\omega(r)\mathcal{W}_\omega(t)Z_0 &= \mathfrak{N}U_\omega(r)(U_1(t)Z_0, U_2(t)Z_0, U_3(t)Z_0, \mathcal{K}\mathcal{N}U_4(t)\mathfrak{K}Z_0)\\ &= (U_1(t+r)Z_0, U_2(t+r)Z_0, U_3(t+r)Z_0, \mathcal{N}\int_{r-s}^{r}\theta(t+\xi)d\xi).\end{aligned}$$

In order to derive (26), we must consider more precisely the fourth component

$$\mathcal{N}\int_{r-s}^{r}\theta(t+\xi)d\xi = \mathcal{N}\int_{t+r-s}^{r}\theta(\xi)d\xi$$
$$= \begin{cases}\mathcal{N}[\int_0^{r+t} U_3(\xi)\mathfrak{K}Z_0 d\xi + (t+r-s)\mathcal{K}q_0], & t+r-s<0\\ \mathcal{N}\int_0^{r+t} U_3(\xi)Z_0 d\xi, & t+r-s\geq 0\end{cases} = \mathcal{N}U_4(t+r)\mathfrak{K}Z_0.$$

Consequently, (26) holds true.

It is easy to see that $\mathfrak{N} \in \mathfrak{L}(H_\omega, \mathcal{V}_\omega)$ and $\mathfrak{K} \in \mathfrak{L}(\mathcal{V}_\omega, H_\omega)$. This entails that for any $Z \in H_\omega$,

$$\begin{aligned}\lim_{t\to 0}\|\mathcal{W}_\omega(t)Z - Z\|_{\mathcal{V}_\omega} &= \lim_{t\to 0}\|\mathfrak{N}U_\omega(t)\mathfrak{K}Z - \mathfrak{N}\mathfrak{K}Z\|_{\mathcal{V}_\omega}\\ &\leq \lim_{t\to 0}\|\mathfrak{N}\|_{\mathfrak{L}(H_\omega,\mathcal{V}_\omega)}\|U_\omega(t)\mathfrak{K}Z - \mathfrak{K}Z\|_{H_\omega} = 0.\end{aligned}$$

Thus, the semigroup $\mathcal{W}_\omega(t)$ is strongly continuous.

Now we will describe the generator of this semigroup. It follows from property (23) that for any $Z \in \mathcal{D}(\mathbb{D}_\omega)$,

$$\lim_{t\to 0}\frac{\mathcal{W}_\omega(t)Z - Z}{t} = \mathfrak{N}\lim_{t\to 0}\frac{U_\omega(t)\mathfrak{K}Z - \mathfrak{K}Z}{t} = \mathfrak{N}\mathbb{A}_\omega\mathfrak{K}Z = \mathbb{D}_\omega Z.$$

The lemma is proved.

Now we establish the well-posedness of problem (25).

Lemma 2. *Assume that conditions* (10)–(12) *hold true. Then problem* (25) *generates the dynamical system* $(\Sigma_\omega(t), \mathcal{V}_\omega)$. *Its evolution operator has the form* $\Sigma_\omega(t) = \mathfrak{N}S_\omega(t)\mathfrak{K}$, *where* $S_\omega(t)$ *is the evolution operator of problem* (14)–(17) *with* $\eta(s) = \kappa e^{-s}$.

P r o o f. For any $Z_0 = (u_0, u_1, \theta_0, q_0) \in \mathcal{V}_\omega$,

$$Z(t) = \Sigma_\omega(t)Z_0 = \mathfrak{N}S_\omega(t)\mathfrak{K}Z_0 = \mathfrak{N}U_\omega(t)\mathfrak{K}Z_0$$

$$+\int_0^t \mathfrak{N}U_\omega(t-\xi)\mathfrak{K}f(Z(\xi))d\xi = \mathcal{W}_\omega(t)Z_0 + \int_0^t \mathcal{W}_\omega(t-\xi)f(Z(\xi))d\xi,$$

where $t \in [0,T]$ for any $0 < T < \infty$. Thus, the function $Z(t) = \Sigma_\omega(t)Z_0 \in C(0,T;\mathcal{V}_\omega)$ is a mild solution to (25). The lemma is proved.

Now we are ready to state the existence result for the attractor of system (20).

Lemma 3. *Let conditions* (10)–(13) *hold true. Then for any $\omega > 0$ the dynamical system $(\Sigma_\omega(t), \mathcal{V}_\omega)$, generated by* (25)*, possesses a compact global attractor $\mathfrak{Q}_\omega$.*

P r o o f. It is easy to see that

$$\|\Sigma_\omega(t)(u_0,u_1,\theta_0,q_0)\|_{\mathcal{V}_\omega} \le \|\mathfrak{N}\|_{\mathfrak{L}(H_\omega,\mathcal{V}_\omega)}\|S_\omega(t)(u_0,u_1,\theta_0,\tau_0)\|_{H_\omega}.$$

Consequently, the dynamical system $(\Sigma_\omega(t), \mathcal{V}_\omega)$ possesses an absorbing ball. Consider the set

$$\mathcal{K} = \{\bigcup_{Y_0\in\mathfrak{A}_\omega}\bigcup_{t\in\mathbb{R}} \mathfrak{N}S_\omega(t)Y_0\} = \mathfrak{N}\{\bigcup_{Y_0\in\mathfrak{A}_\omega}\bigcup_{t\in\mathbb{R}} S_\omega(t)Y_0\}. \tag{27}$$

Since the operator $\mathfrak{N}$ is bounded and the set $\{\bigcup_{Y_0\in\mathfrak{A}_\omega}\bigcup_{t\in\mathbb{R}} S_\omega(t)Y_0\}$ is compact in H_ω, set (27) is compact in $\mathcal{V}_\omega$. Let $B \in \mathcal{V}_\omega$ be a positively invariant set, i.e., $\Sigma_\omega(t)B \subset B$. Then

$$\lim_{t\to+\infty}\sup_{z\in B}\operatorname{dist}_{\mathcal{V}_\omega}(\Sigma_\omega(t)z,\mathcal{K}) \le \lim_{t\to+\infty}\sup_{y\in G}\operatorname{dist}_{\mathcal{V}_\omega}[\mathfrak{N}S_\omega(t)y,\mathfrak{N}\tilde{y}],$$

where

$$G = \{y = \mathfrak{K}z : z \in B\} \subset H_\omega$$

is a bounded set in H_ω, $\tilde{y}$ is the element on which the minimal distance in H_ω from the point $S_\omega(t)y$ to the set $\mathfrak{A}_\omega$ is reached. Then, obviously,

$$\lim_{t\to+\infty}\sup_{y\in B}\operatorname{dist}_{\mathcal{V}_\omega}(\Sigma_\omega(t)y,\mathcal{K}) \le C\lim_{t\to+\infty}\sup_{z\in G}\operatorname{dist}_{H_\omega}(S_\omega(t)y,\mathfrak{A}_\omega) = 0.$$

The above estimate implies that the dynamical system $(\Sigma_\omega(t), \mathcal{V}_\omega)$ is asymptotically compact. This property together with the existence of an absorbing ball is a necessary condition for the existence of a compact global attractor $\mathfrak{Q}_\omega$ (see, e.g., [12, 13]).

Now we study the relation between problems (20), (21) and initial statement (3). Let us introduce the operator $\mathfrak{F} : \mathcal{V}_\omega \to X_\omega$

$$\mathfrak{F} = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & \nabla \end{pmatrix}.$$

It is well known (see [14]) that

$$[L^2(\Omega)]^2 = E_1 \oplus E_2, \tag{28}$$

where

$$E_1 = \{\nabla\phi : \phi \in H_0^1(\Omega)\} \quad \text{and} \quad E_2 = \{\psi \in [L^2(\Omega)]^2 : (\mathrm{div}\psi, \phi) = 0, \phi \in H_0^1(\Omega)\}.$$

By virtue of (28), the initial condition, the problem (3) is subjected to, can be split as $\mathbf{q}_0 = \nabla q_0 + p_0$, where $q_0 \in H_0^1(\Omega)$ and $p_0 \in E_2$. Therefore, we can define the operator $P_1 : [L^2(\Omega)]^2 \to H_0^1(\Omega)$ by the formula $P_1\mathbf{q}_0 = -(-\Delta)^{-1}\mathrm{div}\mathbf{q}_0 = q_0$, where $-\Delta$ is the Laplace operator with the Dirichlet boundary conditions and the projector $P_2 : [L^2(\Omega)] \to E_2$ as $P_2\mathbf{q}_0 = p_0$. Introduce the operators $\mathfrak{P}_1 : X_\omega \to \mathcal{V}_\omega$ and $\mathfrak{P}_2 : X_\omega \to X_\omega$ defined by the formulas

$$\mathfrak{P}_1 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & P_1 \end{pmatrix}, \qquad \mathfrak{P}_2 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & P_2 \end{pmatrix}.$$

The following properties of these operators can be easily checked by the straightforward calculations.

Proposition 3. *For any $Z \in \mathcal{D}(\mathbb{B}_\omega)$,*

$$[\mathfrak{F}\mathfrak{N}\mathbb{A}_\omega\mathfrak{K}\mathfrak{P}_1 + \frac{1}{\omega}\mathfrak{P}_2]Z = \mathbb{B}_\omega Z. \tag{29}$$

Moreover,

$$\mathfrak{F}\mathfrak{P}_1 + \mathfrak{P}_2 = I \in \mathfrak{L}(X_\omega), \tag{30}$$

and for any $Z \in \mathcal{V}_\omega$,

$$\mathfrak{P}_1\mathfrak{F}Z = Z. \tag{31}$$

Define the operator

$$\begin{aligned} \mathfrak{S}_\omega(t)(u_0, u_1, \theta_0, \mathbf{q}_0) &= (0, 0, 0, e^{-\frac{t}{\omega}}p_0) + \mathfrak{F}\Sigma_\omega(t)(u_0, u_1, \theta_0, q_0) \\ &= [e^{-\frac{t}{\omega}}\mathfrak{P}_2 + \mathfrak{F}\Sigma_\omega(t)\mathfrak{P}_1](u_0, u_1, \theta_0, \mathbf{q}_0) \end{aligned} \tag{32}$$

in the space X_ω. We are in position to show that $\mathfrak{S}_\omega(t)$ is the evolution operator of problem (3)–(4).

Lemma 4. *The operator $\mathbb{B}_\omega$ is the generator of the C_0-semigroup $\tilde{U}_\omega(t) = \mathfrak{F}\mathfrak{N}U_\omega(t)\mathfrak{K}\mathfrak{P}_1 + e^{-\frac{t}{\omega}}\mathfrak{P}_2$ on the phase space X_ω.*

P r o o f. First, we prove that $\tilde{U}_\omega(t)$ is the generator of the C_0-semigroup on the space $\mathcal{V}_\omega$. To this end, we check the semigroup properties. We obtain by (30) and (31)

$$\tilde{U}_\omega(0) = \mathfrak{F}\mathcal{W}_\omega(0)\mathfrak{P}_1 + \mathfrak{P}_2 = \mathfrak{F}\mathfrak{P}_1 + \mathfrak{P}_2 = I$$

and

$$\begin{aligned}\tilde{U}_\omega(r)\tilde{U}_\omega(t) &= \mathfrak{F}\mathcal{W}_\omega(r)\mathfrak{P}_1\mathfrak{F}\mathcal{W}_\omega(t)\mathfrak{P}_1 + e^{-\frac{r+t}{\omega}}\mathfrak{P}_2 \\ &= \mathfrak{F}\mathcal{W}_\omega(r+t)\mathfrak{P}_1 + e^{-\frac{r+t}{\omega}}\mathfrak{P}_2 = \tilde{U}_\omega(r+t).\end{aligned}$$

It is easy to see that $\mathfrak{F} \in \mathfrak{L}(X_\omega, \mathcal{V}_\omega)$. Consequently, for any $Z \in X_\omega$,

$$\lim_{t\to 0} \|\tilde{U}_\omega(t)Z - Z\|_{X_\omega}$$
$$= \lim_{t\to 0}[\|\mathfrak{F}\|_{\mathfrak{L}(X_\omega,\mathcal{V}_\omega)}\|\mathcal{W}_\omega(t)\mathfrak{P}_1 Z - \mathfrak{P}_1 Z\|_{\mathcal{V}_\omega} + \|e^{-\frac{t}{\omega}}\mathfrak{P}_2 Z - \mathfrak{P}_2 Z\|_{X_\omega}] = 0.$$

Therefore the semigroup $\tilde{U}_\omega(t)$ is strongly continuous.

Now we describe the generator of the semigroup. It follows from (29) that for any $Z \in \mathcal{D}(\mathbb{B}_\omega)$,

$$\begin{aligned}\lim_{t\to 0}\frac{\tilde{U}_\omega(t)Z - Z}{t} &= \mathfrak{F}\lim_{t\to 0}\frac{\mathcal{W}_\omega(t) - I}{t}\mathfrak{P}_1 Z + \lim_{t\to 0}\frac{e^{-\frac{t}{\omega}} - I}{t}\mathfrak{P}_2 Z \\ &= [\mathfrak{F}\mathfrak{N}\mathbb{A}_\omega\mathfrak{K}\mathfrak{P}_1 + \frac{1}{\omega}\mathfrak{P}_2]Z = \mathbb{B}_\omega Z.\end{aligned}$$

The lemma is proved.

Now we are in position to prove the well-posedness result for (18).

Lemma 5. *Assume that conditions* (10)–(12) *hold true. Then* (18) *generates the nonlinear dynamical system* $(\mathfrak{S}_\omega(t), X_\omega)$ *with the evolution operator defined in* (32)*.*

P r o o f. For any $Z_0 = (u_0, u_1, \theta_0, \mathbf{q}_0) \in X_\omega$,

$$Z(t) = \mathfrak{S}_\omega(t)Z_0 = \mathfrak{F}\Sigma_\omega(t)\mathfrak{P}_1 Z_0 + e^{-\frac{t}{\omega}}\mathfrak{P}_2 Z_0 = \tilde{U}_\omega(t)Z_0 + \int_0^t \tilde{U}_\omega(t-\xi)f(Z(\xi))d\xi,$$

where $t \in [0, T]$ for any $0 < T < \infty$. Therefore the function $Z(t) = \mathfrak{S}_\omega(t)Z_0 \in C(0, T; X_\omega)$ is a mild solution to problem (18). The lemma is proved.

It follows from the above arguments that $(\mathfrak{S}_\omega(t), X_\omega)$ is a dynamical system possessing the compact global attractor $\mathfrak{U}_\omega = \{\mathfrak{F}\mathfrak{Q}_\omega\}$.

By the definition, the fractal dimension of a set is the value

$$\dim_f \mathcal{A} = \overline{\lim_{\varepsilon\to 0}} \frac{\ln n(\mathcal{A},\varepsilon)}{\ln 1/\varepsilon},$$

where $n(\mathcal{A},\varepsilon)$ is the minimal number of closed balls with radius ε covering the compact set $\mathcal{A}$. Obviously,

$$\dim_f \mathfrak{Q}_\omega = \dim_f \mathfrak{U}_\omega. \tag{33}$$

Thus, to show the finite dimensionality of the attractor of the dynamical system $(\mathfrak{S}_\omega(t), X_\omega)$ it remains to prove the lemma bellow.

Lemma 6. *Let* (10)–(12) *hold. Then the attractor* $\mathfrak{Q}_\omega$ *of the dynamical system* $(\Sigma_\omega(t), \mathcal{V}_\omega)$ *generated by problem* (25) *has the finite fractal dimension.*

P r o o f. Consider the restriction of $\mathfrak{N}$ onto the attractor $\mathfrak{A}_\omega$, $\mathcal{P} = \mathfrak{N}|_{\mathfrak{A}_\omega}$. Let us show that $\mathcal{P}\mathfrak{A}_\omega \subset \mathfrak{Q}_\omega$. If $Y \in \mathfrak{A}_\omega$, then there exists a whole trajectory $Y(t) = (u(t), u_t(t), \theta(t), \tau(t)) \in \mathfrak{A}_\omega$ passing through this point. Then $\mathcal{P}Y(t)$ is a bounded whole trajectory passing through the point $X = \mathcal{P}Y$, i.e., X belongs to the attractor $\mathfrak{Q}_\omega$. On the contrary, let $Z = (u, \mathrm{v}, \theta, q) \in \mathfrak{Q}_\omega$, then $Y = \mathfrak{K}Z \in \mathfrak{A}_\omega$ $\mathcal{P}Z = Y$, i.e., $\mathfrak{Q}_\omega \subset \mathcal{P}\mathfrak{A}_\omega$. Therefore, $\mathcal{P}\mathfrak{A}_\omega = \mathfrak{Q}_\omega$, and the mapping $\mathcal{P}$ is continuous. Consequently,

$$\dim_f \mathfrak{Q}_\omega \le \dim_f \mathfrak{A}_\omega < \infty,$$

and the lemma is proved.

Now we will establish the upper semicontinuity of the family of attractors $\{\mathfrak{U}_\omega\}$ with respect to the parameter ω and show that problem (3)–(4) is a singular perturbation of the classical thermoelastic Mindlin–Timoshenko problem (19).

Theorem 2. *Let the assumptions* (6)–(8), (10)–(13) *hold. Then problem* (3)–(4) *generates the dynamical system* $(\mathfrak{S}_\omega(t), X_\omega)$, *where the operator* $\mathfrak{S}_\omega(t)$ *is defined by formula* (32). *For any* $\omega > 0$ *the dynamical system* $(\mathfrak{S}_\omega(t), X_\omega)$ *possesses a compact global finite dimensional attractor* $\mathfrak{U}_\omega$. *The family of attractors* $\{\mathfrak{U}_\omega\}$ *is upper semicontinuous at zero , i.e.,*

$$\sup_{y\in\mathfrak{U}_\omega} \mathrm{dist}_{X_\omega}(y, \mathfrak{B}_0) \to 0, \quad \omega \to 0,$$

where

$$\mathfrak{B}_0 = \left\{ y = \begin{bmatrix} u_0 \\ u_1 \\ \theta_0 \\ -\nabla\theta_0 \end{bmatrix} : \begin{bmatrix} u_0 \\ u_1 \\ \theta_0 \end{bmatrix} \in \mathfrak{A} \right\}.$$

Here $\mathfrak{A}$ *is a compact global attractor of the dynamical system generated by problem* (19) *on the space* $[H_0^1(\Omega)]^3 \times [L^2(\Omega)]^2 \times L^2(\Omega)$.

P r o o f. The existence of the dynamical system and the compact global attractor was proved in Lemmas 1–5. The finite dimensionality of the attractor follows from Lemma 6 and (33).

Since

$$\|\frac{1}{\omega}\nabla \int_0^\infty e^{-\frac{s}{\omega}}(\tau-\theta)ds\| \le \omega\left|\nabla\tau_t\right| \to 0 \;\; \omega \to 0,$$

it follows from Theorem 1 that

$$\lim_{\omega\to 0} \sup_{y\in\mathfrak{U}_\omega} \mathrm{dist}_{X_\omega}(y, \mathfrak{B}_0) = \lim_{\omega\to 0} \sup_{z\in\mathfrak{A}_\omega} \mathrm{dist}_{\mathcal{V}_\omega}(\mathfrak{N}z, \mathfrak{N}\mathfrak{K}\mathfrak{P}_1\mathfrak{B}_0) = 0.$$

The theorem is proved.

References

[1] *D.S. Chandrasekharaiah*, Hyperbolic Thermoelasticity: A Review of Recent Literature. — *Appl. Mech. Rev.* **51** (1998), 705–729.

[2] *J. Lagnese*, Boundary Stabilization of Thin Plates. Philadelphia: SIAM, 1989.

[3] *P. Schiavone and R.J. Tait*, Thermal Effects in Mindlin-Type Plates. — *Q. Jl. Mech. Appl. Math.* **46** (1993), 27–39.

[4] *J.E. Muñoz Rivera and R. Racke*, Global Stability for Damped Timoshenko Systems. — *Disc. Cont. Dyn. Sys.* **9** (2003), 1625–1639.

[5] *J.E. Muñoz Rivera and R. Racke*, Mildly Dissipative Nonlinear Timoshenko Systems — Global Existence and Exponential Stability. — *J. Math. Anal. Appl.* **276** (2002), No. 1, 248–278.

[6] *H.D. Fernández Sare and R. Racke*, On the Stability of Damped Timoshenko Systems: Cattaneo Versus Fourier Law. — *Arch. Rational Mech. Anal.* **194** (2009), 221–251.

[7] *S.A. Messaoudi, M. Pokojovy, and B. Said-Houari*, Nonlinear Damped Timoshenko Systems with Second Sound — Global Existence and Exponential Stability. — *Math. Meth. Appl. Sci.* **32** (2009), No. 5, 505–534.

[8] *I. Chueshov and I. Lasiecka*, Global Attractors for Mindlin–Timoshenko Plates and for Their Kirchhoff Limits. — *Milan J. Math.* **74** (2006), 117–138.

[9] *T. Fastovska*, Upper Semicontinuous Attractor for a 2D Mindlin–Timoshenko Thermoelastic Model with Memory. — *Commun. Pure Appl. Anal.* **6** (2007), No. 1, 83–101.

[10] *T. Fastovska*, Upper Semicontinuous Attractor for a 2D Mindlin–Timoshenko Thermo-Viscoelastic Model with Memory. — *Nonlinear Analysis TMA* **71** (2009), No. 10, 4833–4851.

[11] *M.E. Gurtin and A.C. Pipkin*, A General Theory of Heat Conduction with Finite Wave Speeds. — *Arch. Rational Mech. Anal.* **31** (1968), 113–126.

[12] *I.D. Chueshov*, Introduction to the theory of infinite-dimensional dissipative systems. Acta, Kharkov, 1999. (Russian). (Engl. transl.: Acta, Kharkov, 2002).

[13] *R. Temam*, Infinite-Dimensional Dynamical Systems in Mechanics and Physics. Springer, New York (1988).

[14] *S. Jiang and R. Racke*, Evolution Equations in Thermoelasticity. π Monographs Surveys Pure Appl. Math. **112**, Chapman&Hall/CRC, Boca Raton, 2000.